

# What Do We Learn about Confinement from the Seiberg-Witten Theory \*

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## Abstract

The confinement scenario in  $N = 2$  supersymmetric gauge theory at the monopole point is reviewed. Basic features of this  $U(1)$  confinement are contrasted with those we expect in QCD. In particular, extra states in the hadron spectrum and non-linear Regge trajectories are discussed. Then another confinement scenario arising on Higgs branches of the theory with fundamental matter is also reviewed. Peculiar properties of the Abrikosov–Nielsen–Olesen string on the Higgs branch lead to a new confining regime with the logarithmic suppression of the linear rising potential. Motivations for a search for tensionless strings are proposed.

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# 1 Introduction

Ideas of electromagnetic duality [1] lead to a dramatic breakthrough in our understanding of the dynamics of strongly coupled supersymmetric gauge theories. Particularly spectacular results were obtained by Seiberg and Witten in  $\mathcal{N} = 2$  supersymmetry where the low energy effective Lagrangians were found exactly [2, 3].

One of the most important physical outcomes of the Seiberg–Witten theory is the demonstration of confinement of charges via the monopole condensation. The scenario for confinement as a dual Meissner effect was proposed by Mandelstam and 't Hooft many years ago [4]. However, because the dynamics of monopoles is hard to control in non-supersymmetric gauge theories this picture of confinement remained as an unjustified qualitative scheme.

The breakthrough in this direction was made by Seiberg and Witten [2, 3]. Using the holomorphy imposed by  $\mathcal{N} = 2$  supersymmetry they showed that the condensation of monopoles to really occurs near the monopole point on the modular space of the theory once  $\mathcal{N} = 2$  supersymmetry is broken down to  $\mathcal{N} = 1$  one by the mass term for the adjoint matter.

In this talk I will review the confinement scenario in the Seiberg–Witten theory underlining the basic features of this  $U(1)$  confinement which distinguish it from those we expect in QCD.

In particular, I am going to discuss extra hadron states arising in the Seiberg–Witten theory which we do not expect in QCD or in  $\mathcal{N} = 1$  supersymmetric QCD.

Then I focus on Abrikosov–Nielsen–Olesen (ANO) strings [5] which are responsible for the confinement near the monopole point. We will see that these strings turn out to be too "thick" and cannot be described by the standard string theory approximation of long and infinitely thin strings. The result of this is that Regge trajectories become linear only at very large values of spin  $j$ .

In the second part of my talk I will review another confinement scenario arising in the Seiberg–Witten theory with the fundamental matter: confinement on Higgs branches [3]. Higgs branch represents a limiting case of the superconductor of type I with vanishing Higgs mass. We will see that in this limit ANO vortices becomes logarithmically "thick" [6]. Because of this the confining potential is not linear any longer. It behaves as  $L/\log L$  with the distance between heavy trial charges (monopoles). This peculiar confining regime can occur only in supersymmetric theories.

In the end of my talk I will speculate on the possible ways to avoid at least some of the unwanted features of  $U(1)$  confinement I am discussing in this talk.

## 2 Confinement as a dual Meissner effect

First let me remind the mechanism of confinement suggested by Mandelstam and 't Hooft [4]. Consider an Abelian–Higgs model with the action

$$S_{AH} = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + |\nabla_\mu \varphi|^2 + \lambda(|\varphi|^2 - v^2)^2 \right\}. \quad (2.1)$$

Here  $\varphi$  is a complex scalar field,  $\nabla_\mu = \partial_\mu - in_e A_\mu$ , where  $n_e$  is the electric charge of  $\varphi$ . We assume the weak coupling  $g^2 \ll 1$ . The scalar field in (2.1) develop VEV

$$|\langle \varphi \rangle| = v, \quad (2.2)$$

which breaks  $U(1)$  gauge group. The photon acquires the mass

$$m_\gamma^2 = 2n_e^2 g^2 v^2, \quad (2.3)$$

while the mass of the Higgs boson (one real degree of freedom) is

$$m_H^2 = 4\lambda v^2. \quad (2.4)$$

Now introduce an infinitely heavy trial monopole and anti-monopole in the vacuum of the Abelian Higgs model. We can think of them as of Dirac monopoles or as of 't Hooft–Polyakov monopoles of some underlying non-Abelian gauge theory broken down to  $U(1)$ .

Monopoles has quantized values of magnetic flux  $2\pi n/n_e$ . This magnetic flux cannot be absorbed by the vacuum once there are no dynamical magnetic charges in the theory. On the other hand, magnetic field cannot penetrate into the Higgs vacuum. As a result ANO vortex appears [5] connecting monopole with anti-monopole. This vortex can be viewed as a bubble of an unbroken vacuum inside the Higgs vacuum.

It has  $\varphi = 0$  along the line connecting monopole and anti-monopole and magnetic flux  $2\pi n/n_e$ ,  $n$  — winding number. ANO vortex is a solution of equations of motion for the Abelian–Higgs model (2.1). It corresponds to a non-trivial map from the infinite circle in the plane orthogonal to the axis of

the vortex to the gauge group,  $\pi_1(U(1)) = Z$ . For example, for the winding number  $n = 1$  fields  $\varphi$  and  $A_\mu$  behave at the infinity as

$$\begin{aligned}\varphi &\sim v e^{i\alpha} , \\ A_m &\sim -\varepsilon_{mn} \frac{x_n}{x^2} ,\end{aligned}\tag{2.5}$$

where  $x_n$ ,  $n = 1, 2$  is the distance from the vortex axis in the plane orthogonal to this axis, while  $\alpha$  is the polar angle in this plane.

Because the vortex has a fixed energy per unit length (string tension  $T$ ) the potential between monopole and anti-monopole at large distances  $L$  behaves as

$$V(L) = TL .\tag{2.6}$$

This linear potential means confinement of monopoles.

The ratio of the Higgs mass (2.4) to the photon mass (2.3) is an important parameter characterizing the type of superconductor. If  $m_H > m_\gamma$  we have the type II superconductor. Different vortices interact via repulsive forces. In particular, the tension of the vortex with winding number  $n$  is larger than the sum of tensions of  $n$  vortices with winding numbers  $n = 1$ . Therefore, vortices with higher winding numbers,  $n > 1$  are unstable.

In particular, for  $m_H \gg m_\gamma$  (London limit) the vortex solution can be found in the analytic form. The string tension for this case was calculated by Abrikosov in 1957 and later on re-obtained by Nielsen and Olesen in 1973 [5] in the framework of the relativistic field theory. The result for  $n = 1$  is

$$T = 2\pi v^2 \ln \frac{m_H}{m_\gamma} .\tag{2.7}$$

For the particular interesting case  $m_H = m_\gamma$  vortices satisfy the first order equations. They saturate the Bogomolny bound and their string tension reads [7]

$$T_n = 2\pi v^2 n ,\tag{2.8}$$

where  $n$  is the winding number. In particular, as is clear from (2.8), different vortices do not interact.

In supersymmetric theories the BPS-saturation means that some of SUSY generators act trivially on the vortex solution [8]. The Bogomolny bound (2.8) coincides with the central charge of SUSY algebra. This means that the classical result (2.8) for the vortex string tension remains exact in the quantum theory.

For  $m_H < m_\gamma$  we have type I superconductor. In this case vortices attract each other. In particular, vortices with higher  $n$  are stable [9]. For the case  $m_H \ll m_\gamma$  the vortex solution can be found analytically [6]. The string tension in this case has the form [6]

$$T_n = \frac{2\pi v^2}{\ln m_\gamma/m_H} , \quad (2.9)$$

and does not depend on  $n$  to the leading order in  $\ln m_\gamma/m_H$ .

In the general case the string tension  $T_n$  is a monotonic function of the ratio  $m_H/m_\gamma$  [9]. It reaches its Bogomolny bound (2.8) at  $m_H = m_\gamma$ .

To conclude this section let me point out that the main lesson to learn here is that the condensation of electric charges cause the confinement of monopoles. Vice versa, the condensation of monopoles in the dual Abelian Higgs model leads to the confinement of electric charges.

### 3 Monopole condensation

Now consider  $\mathcal{N} = 2$  gauge theory. Most of all in this talk I will be talking about the simplest case of the theory with  $SU(2)$  gauge group studied in [2]. The simplest version of this theory contains one  $\mathcal{N} = 2$  vector multiplet. This multiplet on the component level consists of the gauge field  $A_\mu^a$ , two Weyl fermions  $\lambda_1^{\alpha a}$  and  $\lambda_2^{\alpha a}$  ( $\alpha = 1, 2$ ) and the complex scalar  $\varphi^a$ , where  $a = 1, 2, 3$  is the color index.

The scalar potential of this theory has a flat direction. Thus the scalar field can develop an arbitrary VEV along this direction breaking  $SU(2)$  gauge group down to  $U(1)$ . We choose  $\langle \varphi^a \rangle = \delta^{a3} \langle a \rangle$ . The complex parameter  $\langle a \rangle$  parameterize the moduli space of the theory (Coulomb branch). The low energy effective theory contains only the photon  $A_\mu = A_\mu^3$  and its superpartners: two Weyl fermions  $\lambda_1^3, \lambda_2^3$  and the complex scalar  $a$ .

The Coulomb branch can be parameterized by the gauge invariant parameter  $u = \langle \varphi^{a2}/2 \rangle$ . It has two singular points  $u = \pm 2\Lambda^2$  ( $\Lambda$  is the scale of gauge theory <sup>2</sup>) where monopole or dyon becomes massless. Near, say, the monopole point ( $u = 2\Lambda^2$ ) the effective low energy theory is dual  $\mathcal{N} = 2$  QED. This means that the theory has light monopole hypermultiplet interacting with the dual photon multiplet in the same way as ordinary charges

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<sup>2</sup>We use the Pauli-Villars regularization scheme here.

interact with the ordinary photon. The action of this dual QED reads

$$S_{eff} = S_g^{eff} + S_m^{eff} . \quad (3.1)$$

Here the action for the gauge field is

$$S_g^{eff} = \int d^4x \left\{ \int d^2\theta d^2\bar{\theta} \frac{1}{g^2} \bar{A}_D A_D + \int d^2\theta \frac{1}{4g^2} W_D^2 + c.c. \right\} , \quad (3.2)$$

where  $A_D$  is the dual chiral  $\mathcal{N} = 1$  field. Its lowest component  $a_D$  goes to zero at the monopole point.  $W_D^\alpha$  ( $\alpha = 1, 2$ ) is the  $\mathcal{N} = 1$  chiral field of the dual photon field strength. Together  $A_D$  and  $W_D$  form  $\mathcal{N} = 2$  vector U(1) multiplet.

The matter-dependent part of the action reads

$$\begin{aligned} S_m^{eff} &= \int d^4x d^2\theta d^2\bar{\theta} \left[ \bar{M} e^V M + \tilde{\bar{M}} e^{-V} \tilde{M} \right] + \\ &+ i \int d^4x d^2\theta \sqrt{2} \tilde{M} A_D M + c.c. . \end{aligned} \quad (3.3)$$

Here  $M, \tilde{M}$  are two chiral fields of the monopole hypermultiplet. The monopole mass (given by  $m_m^2 = 2|a_D|^2$ ) goes to zero at the monopole point  $a_D = 0$ .

Now let us break  $\mathcal{N} = 2$  QED (3.1) down to  $\mathcal{N} = 1$  adding the mass term for the adjoint matter in the microscopic SU(2) Seiberg–Witten theory

$$S_{\text{mass}} = i \int d^4x d^2\theta \mu \Phi^{a2} + c.c. \quad (3.4)$$

where  $\Phi^a$  is the  $\mathcal{N} = 1$  chiral superfield which contains component fields  $\varphi^a$  and  $\lambda_2^{a\alpha}$ . Expressed in terms of  $A_D$  near the monopole point in the effective theory it reads [2]

$$\mu \Phi^{a2} = -\sqrt{2} \xi A_D + \frac{\mu_D}{2} A_D^2 + O(A_D^3) , \quad (3.5)$$

where

$$\begin{aligned} \xi &= 2i \mu \Lambda , \\ \mu_D &= -\frac{27}{4} \mu . \end{aligned} \quad (3.6)$$

The coefficients in (3.6) can be read off the Seiberg–Witten exact solution [2].

Minimizing the superpotential in (3.1),(3.5) with respect to  $M, \tilde{M}$  and  $A_D$  we find that the Coulomb branch shrinks to the point

$$\langle a_D \rangle = 0 ,$$

while

$$\langle \tilde{m} m \rangle = \xi , \quad (3.7)$$

where  $m, \tilde{m}$  are the scalar components of  $M, \tilde{M}$ . Taking into account the  $D$ -term condition in (3.1)

$$|\langle m \rangle| = |\langle \tilde{m} \rangle| \quad (3.8)$$

we get the monopole condensate

$$|\langle m \rangle| = |\langle \tilde{m} \rangle| = \sqrt{|\xi|} . \quad (3.9)$$

The monopole condensation breaks the  $U(1)$  gauge group and ensures confinement of electric charges. ANO strings arising as a result of the monopole condensation connect quarks with anti-quarks (we interpret these states as mesons) or with another quarks with opposite electric charge (we interpret these states as baryons).

Note, that the effective dual QED is in the weak coupling regime at small  $\mu$ . The QED coupling behaves as

$$\frac{8\pi^2}{g^2} \sim -\log \frac{\mu}{\Lambda} . \quad (3.10)$$

Moreover at  $\mu \ll \Lambda$  we can ignore non-Abelian effects. Note, that  $W$ -boson mass  $m_W^2 = 2|a|^2$  is of order of  $\Lambda$  at the monopole point,  $m_W \sim \Lambda$ .

Parameters  $\xi$  and  $\mu_D$  in the mass term perturbation (3.5) play quite different role in the effective QED description of the theory. As it is noted in [10] the linear term in (3.5) is the Fayet–Iliopoulos  $F$ -term which do not break  $\mathcal{N} = 2$  supersymmetry. I will explain this in more details in the next section.

On the contrary, the mass term for  $A_D$  in (3.5) proportional to  $\mu_D$  breaks  $\mathcal{N} = 2$  supersymmetry because it shifts the mass of  $A_D$  away from the photon mass.

## 4 The $U(1)$ confinement versus the QCD-like confinement

Now let me discuss several basic features of the confinement near the monopole point in the Seiberg–Witten theory at small  $\mu$  and contrast them to those we expect in QCD. It is believed that non-supersymmetric Yang-Mills theory is in the same universality class as  $\mathcal{N} = 1$  Yang-Mills theory. The latter can be obtained as a large  $\mu$  limit of the theory under consideration. The reason is that in this limit the adjoint matter becomes heavy and decouples. Therefore, we also expect the QCD-like confinement at large  $\mu$  in the theory at hand. Unfortunately we have no control over the theory in this limit.

Now I will show that the  $U(1)$  confinement in the theory at small  $\mu$  has several important distinctions from what we expect from the theory at large  $\mu$ .

### 4.1 Higher winding numbers

The first problem I would like to talk about arises already in  $SU(2)$  theory [11]. As we discussed before the flux of the ANO vortex is given by its winding number  $n$ , which is an element of  $\pi_1(U(1)) = Z$  (for  $SU(N_c)$  group it is  $\pi_1(U^{N_c-1}(1)) = Z^{N_c-1}$ ). This could produce an extra multiplicity in the hadron spectrum which we do not expect in QCD. In QCD or in the large  $\mu$  limit of the present theory we expect classification of states under the center of the gauge group,  $Z_2$  for  $SU(2)$  rather than  $Z$  ( $Z_{N_c}$  for  $SU(N_c)$ ).

Consider as an example ANO vortex with  $n = 2$  in the effective dual QED at small  $\mu$ . This string connects two quarks with two anti-quarks producing an "exotic" state. Note, that the string with  $n = 2$ , in principle, can be broken by  $W$ -boson pair creation but at low energies we can neglect this process because  $W$ -boson is too heavy at small  $\mu$ ,  $\mu \ll \Lambda$ , ( $m_W \sim \Lambda$ ).

The discussion above of "exotic" states in the hadron spectrum [11] is based on the purely topological reasoning. Now let us consider the dynamical side of the problem. As we have seen in Sect.2, strings with higher  $n$  are stable or unstable depending on the type of the superconductor. Namely, in type I superconductor the energy of the vortex with winding number  $n$  is less than the total energy of  $n$  vortices with winding numbers  $n = 1$ . Therefore vortices with  $n > 1$  are stable and we really have an "exotic" states in the spectrum.



On the contrary, in type II superconductor vortices with  $n > 1$  are unstable against decay into  $n$  vortices with winding numbers  $n = 1$ . Therefore, "exotic" states are unstable and actually in the "real world" at strong coupling might be not observable at all.

Thus the natural question arises: what is the type of superconductivity at the monopole point in the Seiberg–Witten theory. This problem is studied in [12]. Let me briefly present the result.

To the leading order in  $\mu/\Lambda$  we ignore the mass term for  $A_D$  in (3.5) and consider only the linear in  $A_D$  term parameterized by  $\xi$ . As I mentioned before this term is the generalized Fayet–Iliopoulos (FI) term. Let me explain what does this mean. Let us start with  $\mathcal{N} = 1$  QED. In  $\mathcal{N} = 1$  supersymmetric U(1) gauge theory one can add FI term to the action [13] (we call it FI  $D$ -term here)

$$- \xi_3 D , \quad (4.1)$$

where  $D$  is the  $D$ -component of the gauge field. In  $\mathcal{N} = 2$  SUSY theory field  $D$  belongs to the  $SU_R(2)$  triplet together with  $F$ -components of the field  $A_D$ ,  $F_D$  and  $\bar{F}_D$ . Namely, let us introduce the triplet  $F_a$  ( $a = 1, 2, 3$ ) using relations

$$\begin{aligned} D &= F_3 , \\ F_D &= \frac{1}{\sqrt{2}} (F_1 + iF_2) , \\ \bar{F}_D &= \frac{1}{\sqrt{2}} (F_1 - iF_2) . \end{aligned} \quad (4.2)$$

Now the generalized FI-term can be written as

$$S_{FI} = -i \int d^4x \, \xi_a F_a . \quad (4.3)$$

Comparing this with (3.5) we identify

$$\begin{aligned} \xi &= \frac{1}{2} (\xi_1 - i\xi_2) , \\ \bar{\xi} &= \frac{1}{2} (\xi_1 + i\xi_2) . \end{aligned} \quad (4.4)$$

For this reason we call the term linear in  $A_D$  in (3.5) FI  $F$ -term.

It is well known that the  $\mathcal{N} = 1$  QED with FI  $D$ -term has BPS ANO string [8, 14, 15]. The reason for this is the following. After the breakdown

of  $U(1)$  gauge group the photon acquires the mass given by (2.3) ( $v^2 = 2|\xi|$  in (2.3), see (3.9)). The massive vector  $\mathcal{N} = 1$  supermultiplet contains, in particular, one real scalar. This scalar plays the role of the Higgs boson in the Abelian Higgs model (2.1). Thus the BPS condition  $m_H = m_\gamma$  is imposed by supersymmetry.

Now return to  $\mathcal{N} = 2$  supersymmetry and consider dual QED (3.1) with the FI  $F$ -term added. The FI parameter  $\xi^a$  explicitly breaks the  $SU_R(2)$  group. However, the  $\mathcal{N} = 2$  supersymmetry remains unbroken. To see this note that FI term is proportional to  $F$  and  $D$  components of the vector multiplet which transform as a total derivatives under the  $\mathcal{N} = 2$  supersymmetry transformation.

It is clear that the FI  $F$ -term which appears in the Seiberg–Witten theory (see (3.5)) can be obtained by  $SU_R(2)$  rotation from the FI  $D$ -term. Moreover, masses of particles in  $\mathcal{N} = 2$  multiplet do not change under this rotation. In particular, the condition  $m_H = m_\gamma$  stays intact. Hence, the ANO string is BPS-saturated [10, 16, 12]. Its string tension is given by (2.8). For  $n = 1$

$$T = 2\pi(2|\xi|) , \quad (4.5)$$

where we use that  $v^2 = 2|\xi|$ .

To consider the next-to-leading correction in  $\mu/\Lambda$  we switch on the mass term for  $A_D$  parameterized by  $\mu_D$  in the effective theory, see (3.5). It breaks the  $\mathcal{N} = 2$  supersymmetry and split the  $\mathcal{N} = 2$  multiplet. In particular, it breaks the BPS condition  $m_H = m_\gamma$ . The effect of this term is studied in [12]. The result is that the Higgs mass in the effective Abelian Higgs model appears to be less than the photon mass,  $m_H < m_\gamma$  and the theory is driven to the type I superconductivity. The string tension is less than its Bogomolny bound,

$$T < 2\pi(2|\xi|) . \quad (4.6)$$

In particular, in the large  $\mu_D$ -limit  $\mu_D^2 \gg \xi$  the string tension is found analytically <sup>3</sup>[12]:

$$T = \frac{2\pi(2|\xi|)}{\ln(\frac{g|\mu_D|}{2\sqrt{|\xi|}})} . \quad (4.7)$$

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<sup>3</sup>To take this limit we ignore relations (3.6), (3.10) and consider QED (3.1) with the perturbation (3.5) on its own right viewing parameters  $\xi$ ,  $\mu_D$  and  $g^2$  as independent ones, assuming only the weak coupling condition  $g^2 \ll 1$  [12].

As I explained above the result in (4.6) means that we really have an infinite tower of "exotic" hadron states with higher string fluxes near the monopole point at small  $\mu$ .

Of course, if we increase  $\mu$  and approach  $\mu \sim \Lambda$  the unwanted strings with  $n > 1$  become broken by the  $W$ -bosons production. The only string with  $n = 1$  connecting one quark with one anti-quark<sup>4</sup> will probably survive. This string is believed to be responsible for the confinement in  $\mathcal{N} = 1$  SQCD at large  $\mu$ . It is called QCD string. This picture of confinement is proposed in refs.[17, 10] within the brane approach.

However, this scenario is hard to implement in the field theory. One reason for this is that at large  $\mu$  dual QED enters the strong coupling regime and is no longer under control. Another one is probably even more fundamental. The point is that the role of matter fields in the effective QED (3.1) is played by monopoles. As  $\mu$  approaches  $\Lambda$  the inverse mass of the dual photon (2.3) approaches the size of monopole (which is of order of the inverse  $W$ -boson mass  $m_W^{-1} \sim \Lambda^{-1}$ ). Under these conditions we hardly can consider monopoles as a local degrees of freedom and the dual QED effective description breaks down. In particular, we don't have a field theoretical description of  $n = 1$  string in the region of large  $\mu \geq \Lambda$ .

## 4.2 p-strings

Another problem of  $U(1)$  confinement in Seiberg–Witten theory was noticed by Douglas and Shenker [18]. It appears in  $SU(N_c)$  gauge theories at  $N_c \geq 3$ .

Scalar VEV's breaks the gauge group down to  $U(1)^{N_c-1}$ . Hence,  $N_c - 1$  different types of ANO vortices arise each one associated with a particular  $U(1)$  factor. Their string tensions are given by [18]

$$T_p \sim \xi \sin \frac{\pi p}{N_c}, \quad (4.8)$$

where  $p = 1, \dots, N_c - 1$  numerates different  $U(1)$  factors.. They are called  $p$ -strings. The number of strings with different string tensions equals to  $[(N_c - 1)/2]$ .

Therefore it is clear that extra states emerge in the hadron spectrum which we do not expect in QCD [18]. Namely, the number of quark–anti-quark meson states is  $N_c$ . Let me explain this for the case of  $SU(3)$ .

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<sup>4</sup>Or with another quark with the opposite electric charge.

We take three different quarks of  $SU(3)$  as

$$q_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad q_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad q_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (4.9)$$

and choose generators of two  $U(1)$  groups of the broken  $SU(3)$  to be

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (4.10)$$

Thus, three quarks in (4.9) have the following electric charges with respect to two  $U(1)$  groups:

$$(1, 0) ; \quad (-1, 1) ; \quad (0, -1) . \quad (4.11)$$

Now it is clear that  $\tilde{q}_1 q_1$  meson is formed by 1-string,  $\tilde{q}_2 q_2$  meson is formed by 2-string and 1-anti-string and  $\tilde{q}_3 q_3$  mesons is formed by 2-anti-string. In sum we have 3 meson states. Two of them ( $\tilde{q}_1 q_1$  and  $\tilde{q}_3 q_3$ ) are (classically) degenerative while the third one,  $\tilde{q}_2 q_2$  is two times heavier than the first two

In general we have  $[(N_c + 1)/2]$  families of  $\tilde{q}q$  mesons with different masses [18, 10]. Each family contains two mesons (one family contains one meson if  $N_c$  is odd). They are classically degenerative but can split in quantum theory.

Let me illustrate this splitting for the simplest example of  $SU(2)$  gauge group. For  $SU(2)$  we have two classically degenerative mesons. If we decompose the quark as

$$q = q_+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + q_- \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (4.12)$$

then these two mesons are  $\tilde{q}_+ q_+$  and  $\tilde{q}_- q_-$ . Out of these states we can form two different combinations as follows

$$\begin{aligned} \tilde{q}q &= \tilde{q}_+ q_+ + \tilde{q}_- q_- , \\ \tilde{q}\tau_3 q &= \tilde{q}_+ q_+ - \tilde{q}_- q_- . \end{aligned} \quad (4.13)$$

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<sup>5</sup>This is true if we neglect masses of quarks at the ends of strings (see subsection 4.4 for the discussion of this point).

Here  $\tau_3$  is the color matrix. In gauge invariant notation two states in (4.13) are

$$\tilde{q}q, \quad \frac{1}{\sqrt{\varphi^2}} \tilde{q}\varphi q. \quad (4.14)$$

It is clear that classically these states are degenerative but in quantum theory they split. The heavier state acquires a large width and might not be observable at all.

Still we have  $[(N_c + 1)/2]$  different  $\tilde{q}q$  meson states instead of one state we expect in the large  $\mu$  limit.

As it is noted in [18]  $W$ -bosons are charged under different  $U(1)$  factors and therefore provide a coupling between them. Thus, we expect that at large  $\mu$ ,  $\mu \sim \Lambda$  most of  $\tilde{q}q$ -mesons discussed above become unstable and disappear. In fact, it is shown in [10] within the brane approach that as we increase  $\mu$  all  $\tilde{q}q$ -meson disappear except one in which quark is connected with anti-quark by the 1-string. This string is shown [10] to become QCD string of ref.[17] at large  $\mu$ .

Unfortunately, as I mentioned before, we still have no description of this transition in field theory.

$P$ -strings by themselves also survive the large  $\mu$ - limit [10]. They connect  $p$  quarks in the  $p$ -index antisymmetric representation with  $p$ -anti-quarks to form an "exotic" meson. Also they can form baryons.

To conclude this subsection I would like to mention the recent paper [19] in which  $\mathcal{N} = 2$  supersymmetry breaking with the first two Casimir operators is considered for the  $SU(N_c)$  theory. It is claimed that once off-diagonal couplings between different  $U(1)$  factors are taken into account  $p$ -strings in generic case fail to be BPS saturated even in the limit of zero  $\mu_D$ .

### 4.3 Non-linear Regge trajectories

One of the main motivations to consider hadrons as quarks connected by strings in early days of String Theory was the linear behavior of Regge trajectories. However, as I show now the ANO string at the monopole point of the Seiberg–Witten theory does not produce linear Regge trajectories. More precisely Regge trajectories become linear only at very large  $j$ .

Let me first show the linear behavior of Regge trajectories using very elementary classical arguments.

Consider long ANO string (of the length  $L$ ) connecting light quark and

anti-quark <sup>6</sup>. Let this string to rotate around the axis orthogonal to the string with large spin  $j$ . If  $j$  is large enough the problem becomes classical and we can apply classical equations of motion to the rotating string. For the linear potential

$$V(L) = TL \quad (4.15)$$

equation of motion looks like

$$T \sim E\omega^2 L, \quad (4.16)$$

where  $E$  is the mass of the string (we neglect the quark masses)

$$E \sim V(L) = TL, \quad (4.17)$$

while  $\omega$  is the frequency of the angular rotation. Expressing  $\omega$  in terms of  $j$  using  $j \sim E\omega L^2$  we obtain

$$L^2 \sim \frac{j}{T}. \quad (4.18)$$

Using (4.17) again we get that the meson mass is

$$E^2 \sim Tj. \quad (4.19)$$

The mass square is proportional to  $j$  at large  $j$ .

More precisely the spectrum of a free Nambu–Goto string goes as

$$E^2 = E_0^2 + T(j + n). \quad (4.20)$$

Here  $E_0^2$  is the intercept and  $n$  labels the daughter trajectories. At large  $j$ ,  $j \gg 1$  our naive estimate (4.19) gives the same result as the exact string spectrum (4.20).

The string tension  $T$  is given by (4.5) to the leading order in  $\mu$ . Expressed in terms of the photon mass (2.3) (or Higgs mass,  $m_\gamma \simeq m_H$ ) it reads

$$T = \frac{\pi m_\gamma^2}{n_e^2 g^2}. \quad (4.21)$$

We see that in terms of photon mass  $T$  is large in the weak coupling  $g^2 \ll 1$ . This is a typical result for solitonic objects in the semiclassical approximation.

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<sup>6</sup>See next subsection for the discussion on whether quarks can be light near the monopole point.

Now let us discuss the region of validity of (4.19) and (4.20). The string theory result (4.20) assumes the approximation of long and thin strings. The transverse size of the ANO vortex is given by  $1/m_\gamma$ . So we need

$$L^2 \gg \frac{1}{m_\gamma^2} . \quad (4.22)$$

Substituting (4.18) and (4.21) here we get

$$j \gg \frac{1}{g^2} . \quad (4.23)$$

We see that in fact Regge trajectories become linear only at extremely large  $j$ .

The bound (4.23) is rather restrictive. If  $j$  is not that large the transverse size of the vortex becomes of order of its length and the string is not developed. Quark and anti-quark are on the border between the stringy regime and the Coulomb regime. At small  $j$   $\tilde{q}q$ -meson more looks like spherically symmetric soliton rather than a string.

We can also see the breakdown of the string description from the string representation for the ANO vortex. It is developed in [20, 21] and [6] for the cases of strings in the type II superconductor, BPS-strings and strings in the type I superconductor respectively. The common feature of these representations is that the leading term of the world sheet action is the Nambu-Goto term

$$S_{\text{string}} = T \int d^2x \{ \sqrt{g} + \text{higher derivatives} \} , \quad (4.24)$$

where  $g_{ij} = \partial_i x_\mu \partial_j x_\mu$  is the induced metric ( $i, j = 1, 2$ ). Higher derivative corrections in (4.24) contain term important for the string quantization [22, 23], rigidity term [24] etc. These terms contain powers of  $\partial/m_\gamma$ . For thin strings  $m_\gamma \sim m_H \rightarrow \infty$  and these corrections can be neglected in the action (4.24). However, for the ANO vortex in the semiclassical regime  $\partial^2/m_\gamma^2 \sim T/m_\gamma^2 \sim 1/g^2 \gg 1$  (see (4.21)). Hence, higher derivative corrections blow up in (4.24) and the string approximation is no longer acceptable. From the string theory point of view this manifest itself as a "crumpled" string surface [24, 25].

We see that Regge trajectories are not linear in the wide region of spins  $j \lesssim 1/g^2$ . This to be contrasted with perfect linear behavior of Regge tra-

jectories in Nature starting from small  $j$ <sup>7</sup> (for a recent account see lecture [26] given at this School).

It might seem funny that we are trying to compare some properties of Seiberg–Witten theory with experiment. Still I believe that the linear behavior of Regge trajectories is an important feature of confinement in QCD and we have to reproduce it in a theory with QCD-like confinement.

The main property responsible for this ”disadvantage” of the confinement in the monopole point of the Seiberg–Witten theory is the large value of the string tension (4.21). In the conclusion of this talk I will speculate that this problem as well as some others could be resolved if the string were (almost) tensionless.

## 4.4 Heavy quark-anti-quark states

Another unpleasant consequence of the large value of the string tension (4.21) is that hadrons built of quarks appear to be too heavy.

To show this let me summarize qualitatively the low lying hadron spectrum in our theory. First, there are states with color (magnetic) charge screened by the Higgs mechanism. They are monopoles (described by operators  $\widetilde{M}M$ ) and the dual photon with its superpartners. Their masses are of order of  $m_\gamma$  ( $m_H \approx m_\gamma$  at small  $\mu$ ).

Second, there are hadrons built of quarks via the confinement mechanism described above. As an example, consider  $\tilde{q}q$ -meson at  $j \sim 1$ . Its mass is of order of

$$m_{\tilde{q}q} \sim 2m_q + \sqrt{T} , \quad (4.25)$$

where  $m_q$  is the quark mass.

The problem is that the mass in (4.25) is too large (as compared with  $m_\gamma$ ) as I will show below. This means that we have light monopole states and the photon (which we can interpret as glueballs), while states built of quarks are heavy. In contrast, in QCD we have light  $\tilde{q}q$ -states, whereas the candidates for glueballs are much heavier.

First, let us see how small  $m_q$  in (4.25) can be. So far we discussed the pure gauge theory or the theory with very heavy quarks which we used as a probe for the confinement. Now let us introduce one dynamical flavor of the fundamental matter hypermultiplet [3](we call it quark).

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<sup>7</sup> On the contrary, in QCD is hard to talk about linear trajectories at large  $j$  because higher resonances acquire large widths.



The quark mass on the Coulomb branch of  $\mathcal{N} = 2$  theory is given by

$$m_q = m + \frac{a}{\sqrt{2}} , \quad (4.26)$$

where  $m$  is the quark mass parameter in the microscopic theory. Quarks become light near the charge singular point on the Coulomb branch,  $a = -\sqrt{2}m$ . Hence, in order to have light quarks we have to go near the charge singularity.

On the other hand, once we switch on  $\mu$  the Coulomb branch shrinks to three singular points: monopole, dyon and charge ones. As we discuss before, in order to have monopole condensation and quark confinement we have to go to the monopole point.

Now to make quarks light near the monopole point we choose  $m$  to ensure that the charge singularity goes close on the Coulomb branch to the monopole one. The values of  $m$  corresponding to the colliding of these singularities are called Argyres–Douglas (AD) points [27]. In  $SU(2)$  theory these points were studied in [28]. On the Coulomb branch of  $\mathcal{N} = 2$  theory these points flow in the infrared to a non-trivial conformal field theories.

The value of  $m$  at which the monopole singularity collides with the charge one is [28] (there are three such points; we choose one of them corresponding to real  $m_{AD}$ )

$$m_{AD} = \frac{3}{2^{4/3}} \Lambda_1 , \quad (4.27)$$

where  $\Lambda_1$  is the scale of the theory with one flavor,  $\Lambda^4 = m\Lambda_1^3$  at large  $m$ .

The problem, however, is that we cannot go directly to the AD-point (4.27). The point is that the monopole condensate vanishes at the AD-point [29]. This means that the AD-point is the point of the quark deconfinement [29].

Now the question is whether we can go close to the AD-point to make quarks light enough without destroying the monopole condensation and quark confinement. To answer this question let us develop a perturbation theory around AD-point.

In general, the monopole condensate in the theory with one flavor is [29]

$$\langle \widetilde{M} M \rangle = 2i\mu \left( u_m^2 - 2m\Lambda_1^3 \right)^{1/4} , \quad (4.28)$$

where  $u_m$  is the position of the monopole singularity in the  $u$ -plane given by the Seiberg–Witten curve [3]. In particular, in the AD-point  $u_m^{AD} = \frac{4}{3}m_{AD}^2$ . This value makes the monopole condensate in (4.28) vanish.

Now let us take  $m$  close to its AD-value

$$m = m_{AD}(1 + \varepsilon) , \quad (4.29)$$

where  $\varepsilon \ll 1$ . Then extracting  $u_m$  from the Seiberg–Witten curve near AD point [29] we get

$$\langle \widetilde{M}M \rangle \sim \mu \Lambda_1 \varepsilon^{1/4} . \quad (4.30)$$

The value of the monopole condensate (4.30) set the scale of our effective Abelian Higgs model. In particular, the photon mass and ANO string tension is given by Eqs. (2.3) and (2.8), where the Higgs VEV  $v^2$  is identified with the monopole condensate (4.30).

Now let us see if we can make quark mass in (4.25) to be small as compared to the string scale  $T^{1/2} \sim \langle \widetilde{M}M \rangle^{1/2}$ . According to ref.[28] the anomalous dimension of  $m_q$  in (4.26) is one, while the anomalous dimension of  $m - m_{AD}$  is 4/5. Using this we conclude that

$$m_q \sim \Lambda_1 \varepsilon^{5/4} . \quad (4.31)$$

From (4.30) and (4.31) we see that we can always make quarks lighter than the string scale  $T^{1/2}$  if we take  $\mu$  not too small,  $\mu \gg \Lambda_1 \varepsilon^{9/4}$ . Of course, we still keep  $\mu \ll \Lambda_1$  to ensure the validity of our dual QED description.

Unfortunately, this does not solve the problem of heavy  $\tilde{q}q$ -states. As we already mentioned, the string scale is much larger or at least of the same order as the photon mass, (see (4.21)). Therefore, the mass of  $\tilde{q}q$ -meson in (4.25) is much larger or of the same order as the photon and monopole masses in contrast with our expectations about the theory with QCD-like confinement.

## 5 Confinement on Higgs branches

In this section I will consider another confinement scenario in the Seiberg–Witten theory: confinement on the Higgs branch in the theory with matter. I will talk about  $SU(2)$  theory with  $N_f = 2$  flavors of fundamental matter [3]. The  $N_f = 2$  case is the simplest case of the theory with Higgs branches (for  $N_f = 1$  we do not have Higgs branches).

## 5.1 Review of Higgs branches in SU(2) theory

Let us introduce  $N_f = 2$  fundamental matter hypermultiplets in the  $\mathcal{N} = 2$  SU(2) gauge theory. In terms of  $\mathcal{N} = 1$  superfields matter dependent part of the microscopic action looks like

$$S_{\text{matter}} = \int d^4x d^2\theta d^2\bar{\theta} \left[ \bar{Q}_A e^V Q^A + \bar{\tilde{Q}}^A e^V \tilde{Q}_A \right] + \\ + i \int d^4x d^2\theta \left[ \sqrt{2} \tilde{Q}_A \frac{\tau^a}{2} Q^A \Phi^a + m \tilde{Q}_A Q^A \right] + \text{c.c.} \quad (5.1)$$

Here  $Q^{kA}, \tilde{Q}_{Ak}$  are matter chiral fields,  $k = 1, 2$  and  $A = 1, \dots, N_F$ , while  $V$  is the vector superfield. Thus we have 16 real matter degrees of freedom for  $N_f = 2$ .

Consider first the limit of large  $m$ . In this limit the three singularities on the Coulomb branch are easy to understand. Two of them correspond to monopole and dyon singularities of the pure gauge theory. Their positions on the Coulomb branch are given by [3]

$$u_{m,d} = \pm 2m\Lambda_2 - \frac{1}{2}\Lambda_2^2, \quad (5.2)$$

where  $u = \frac{1}{2}\langle\varphi^{a^2}\rangle$  and  $\Lambda_2$  is the scale of the theory with  $N_f = 2$ . In the large  $m$  limit  $u_{m,d}$  are approximately given by their values in the pure gauge theory  $u_{m,d} \simeq \pm 2m\Lambda_2 = \pm 2\Lambda^2$ , where  $\Lambda$  is the scale of  $N_f = 0$  theory.

The third singularity corresponds to the point where charge becomes massless. Let us decompose matter fields as

$$Q^{kA} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^k Q_+^A + \begin{pmatrix} 0 \\ 1 \end{pmatrix}^k Q_-^A. \quad (5.3)$$

From the superpotential in (5.1) we see that the  $Q_+$  becomes massless at

$$a = -\sqrt{2}m. \quad (5.4)$$

The singular point  $a = +\sqrt{2}m$  is gauge equivalent to the one in (5.4). In terms of variable  $u$  (5.4) reads

$$u_c = m^2 + \frac{1}{2}\Lambda^2. \quad (5.5)$$

Strictly speaking, we have  $2 + N_f = 4$  singularities on the Coulomb branch. However two of them coincides for the case of two flavors of matter with the same mass.

The effective theory on the Coulomb branch near charge singularity (5.4) is given by  $\mathcal{N} = 2$  QED with light matter fields  $Q_+^A, \tilde{Q}_{+A}$  (8 real degrees of freedom) as well as the photon multiplet.

The charge singularity (5.4),(5.5) is the root of the Higgs branch [3]. To find this branch let us write down  $D$ -term and  $F$ -term conditions which follow from (5.1).  $D$ -term conditions are

$$Q^{kA}\bar{Q}_{A\ell} + \bar{\tilde{Q}}^{kA}\tilde{Q}_{A\ell} = 0, \quad (5.6)$$

while  $F$ -term conditions give (5.4) as well as

$$Q^{kA}\tilde{Q}_{A\ell} = 0. \quad (5.7)$$

Eqs. (5.6),(5.7) have nontrivial solutions for  $N_f \geq 2$ . These solutions determines VEV's for scalar components  $q^{kA}, \tilde{q}_{Ak}$  of fields  $Q^{kA}, \tilde{Q}_{Ak}$ . Dropping heavy components  $q_-$  according to decomposition (5.3) and introducing the  $SU_R(2)$  doublet  $q^{fA}$  as

$$\begin{aligned} q^{1A} &= q_+^A, & q^{2A} &= \bar{\tilde{q}}_+^A, \\ \bar{q}_{A1} &= \bar{q}_A^+, & \bar{q}_{A2} &= -\tilde{q}_A^+, \end{aligned} \quad (5.8)$$

we can rewrite three real conditions in (5.6),(5.7) as

$$\bar{q}_{Ap}(\tau^a)_f^p q^{fA} = 0, \quad a = 1, 2, 3. \quad (5.9)$$

Eq.(5.9) together with the condition (5.4) determines the Higgs branch (manifold with  $\langle q \rangle \neq 0$ ) which touches the Coulomb branch at the point (5.4).

The low energy theory for boson fields near the root of the Higgs branch looks like

$$S_{\text{boson}}^{\text{root}} = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + \bar{\nabla}_\mu \bar{q}_{Af} \nabla_\mu q^{fA} + \frac{g^2}{8} [\text{Tr } \bar{q} \tau^a q]^2 \right\}, \quad (5.10)$$

where trace is calculated over flavor and  $SU_R(2)$  indices. Here  $\nabla_\mu = \partial_\mu - in_e A_\mu$ ,  $\bar{\nabla}_\mu = \partial_\mu + in_e A_\mu$ , the electric charge  $n_e = 1/2$  for fundamental matter fields.

This is an Abelian Higgs model with last interaction term coming from the elimination of  $D$  and  $F$  terms. The QED coupling constant  $g^2$  is small near the root of the Higgs branch. We include 8 real matter degrees of freedom  $q^{fA}$  in the theory (5.10) according to the identification (5.8). The

rest of matter fields  $q^A, \tilde{q}_A^-$  (another 8 real degrees of freedom) acquire a large mass  $2m$  and can be dropped out. The effective theory (5.10) is correct on the Coulomb branch near the root of the Higgs branch (5.4) or on the Higgs branch not far away from the origin  $\langle q \rangle = 0$ .

It is clear that the last term in (5.10) is zero on the fields  $q$  which satisfy constraint (5.9). This means that moduli fields which develop VEV's on the Higgs branch are massless, as it should be. The other fields acquire mass of order  $\langle \bar{q}q \rangle^{1/2}$ . It turns out that there are four real moduli fields  $q$  (out of 8) which satisfy the constraint (5.9) [3]. They correspond to the lowest components of one hypermultiplet.

We can parameterize them as

$$q^{f\dot{A}}(x) = \frac{1}{\sqrt{2}} \sigma_{\alpha}^{f\dot{A}} \phi_{\alpha}(x) e^{i\alpha(x)}. \quad (5.11)$$

Here  $\phi_{\alpha}(x), \alpha = 1 \dots 4$  are four real moduli fields. It is clear that fields (5.11) solve (5.9). The common phase  $\alpha(x)$  in (5.11) is the U(1) gauge phase. Once  $\langle \phi_{\alpha} \rangle = v_{\alpha} \neq 0$  on the Higgs branch the U(1) group is broken and  $\alpha(x)$  is eaten by the Higgs mechanism. Say, in the unitary gauge  $\alpha(x) = 0$ . In the next subsection we consider vortex solution for the model (5.10). Then  $\alpha(x)$  is determined by the behavior of the gauge field at the infinity. Substituting (5.11) into (5.10) we get the bosonic part of the effective theory for the massless moduli fields on the Higgs branch near the origin

$$S_{\text{boson}}^{\text{Higgs}} = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + \bar{\nabla}_{\mu} \bar{q}_{\alpha} \nabla_{\mu} q_{\alpha} \right\}, \quad (5.12)$$

where

$$q_{\alpha}(x) = \phi_{\alpha}(x) e^{i\alpha(x)}. \quad (5.13)$$

Once  $v_{\alpha} \neq 0$  we expect monopoles (they are heavy at  $m \gg \Lambda$ ) to confine via formation of vortices which carry the magnetic flux. The peculiar feature of the theory (5.12) is the absence of the Higgs potential. Therefore, the Higgs phase of the theory in (5.12) is the limiting case of type I superconductor with the vanishing Higgs mass. In the next subsection I will consider the peculiar features of ANO vortices in the model (5.12).

If we increase  $v_{\alpha}^2$  taking  $v_{\alpha}^2 \gtrsim \Lambda^2$  we can integrate out massive photon. Then the effective theory is a  $\sigma$ -model for massless fields  $q_{\alpha}$  which belong to 4-dimensional Hyper-Kähler manifold,  $R^4/Z_2$ . The metric of this  $\sigma$ -model is flat [3, 30], there are, however, higher derivative corrections induced by

instantons [31]. Here we consider region of Higgs branch with  $v_\alpha^2 \ll \Lambda_2$ . This determines the scale of the effective Abelian Higgs model (5.12).  $W$ -bosons and other particles which reflect the non-Abelian structure of the underlying microscopic theory are heavy with masses  $\gtrsim \Lambda_2$  and can be ignored.

To conclude this subsection let us briefly review what happens if we reduce the mass parameter  $m$ . At  $m = \pm\Lambda$  the charge singularity (root of the Higgs branch) collides with the monopole (dyon) singularity, see Eqs.(5.2),(5.5). These are Argyres–Douglas points [27, 28]. At these points mutually non-local degrees of freedom (say, charges and monopoles) becomes massless simultaneously. These points are very interesting from the point of view of the monopole confinement on the Higgs branch. Monopoles become dynamical as we approach Argyres–Douglas point,  $m \rightarrow \Lambda_2$ .

After the collision quantum numbers of particles at singularities change because of monodromies [3]. If we denote quantum numbers as  $(n_m, n_e)_B$ , where  $n_m$  and  $n_e$  are magnetic and electric charges of the state, while  $B$  is its baryon number then at  $m > \Lambda_2$  we have charge, monopole and dyon singularities with quantum numbers

$$(0, 1/2)_1^{*2}, \quad (1, 0)_0, \quad (1, 1)_0. \quad (5.14)$$

The superscript for the charge means that we have two flavors of charges. After charge singularity collides with monopole one (at  $m < \Lambda_2$ ) the quantum numbers of particles at singularities become [32]

$$(1, 0)_0^{*2}, \quad (1, 1/2)_1, \quad (1, 1/2)_{-1}. \quad (5.15)$$

Now monopole  $(1, 0)_0$  condense on the Higgs branch which emerges from the point (5.5), while dyons  $(1, 1/2)_1$  and  $(1, 1/2)_{-1}$  confine because they carry electric charge. At zero mass,  $m = 0$  two dyon singularities in (5.15) coincide (see (5.2)) and the second Higgs branch appears at the point  $u = -1/2 \Lambda_2^2$ . This restores the global symmetry from  $SU(N_f = 2)$  in the massive theory to  $SO(2N_f = 4)$  at  $m = 0$  [3].

## 5.2 ANO string on the Higgs branch

Now let us focus on confinement of monopoles on the Higgs branch and consider ANO vortices in the model (5.12) <sup>8</sup>. This is done in [6].

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<sup>8</sup> Let us take  $m > \Lambda_2$  to avoid confusion with notation of dyon states.

Without loss of generality we take VEV's of  $q_\alpha v_\alpha = (v, 0, 0, 0)$ . Moreover, we drop fields  $q_2, q_3$  and  $q_4$  from (5.12) because they are irrelevant for the purpose of finding classical vortex solutions. Thus, we arrive at the Abelian Higgs model (2.1) with  $\lambda = 0$  and identification  $q_1 = \varphi$ .

Following [6] consider first the model (2.1) with small  $\lambda$ , so that  $m_H \ll m_\gamma$  (see (2.3), (2.4) for  $n_e = 1/2$ ). Then we take the limit  $m_H \rightarrow 0$ .

To the leading order in  $\log m_\gamma/m_H$  the vortex solution has the following structure [6]. The electromagnetic field is confined in a core with the radius

$$R_g^2 \sim \frac{1}{m_\gamma^2} \ln^2 \frac{m_\gamma}{m_H} . \quad (5.16)$$

The scalar field is close to zero inside the core. Instead, outside the core, the electromagnetic field is vanishingly small. At intermediate distances

$$R_g \ll r \ll \frac{1}{m_H} \quad (5.17)$$

( $r$  is the distance from the center of vortex in (1,2) plane) the scalar field satisfy the free equation of motion. Its solution reads [6]

$$\varphi(r) = v \left( 1 - \frac{\ln 1/r m_H}{\ln 1/R_g m_H} \right) . \quad (5.18)$$

At large distances  $r \gg 1/m_H$   $\varphi$  approaches its VEV as  $\varphi - v \sim \exp(-m_H r)$ .

The main contribution to the string tension comes from the logarithmically large region (5.17), where scalar field is given by (5.18). The result for the string tension is [6] (as we already mentioned, see (2.9))

$$T = \frac{2\pi v^2}{\ln m_\gamma/m_H} . \quad (5.19)$$

It comes from the kinetic energy of the scalar field in (2.1) ("surface" energy).

The results in (5.16), (5.19) mean that if we naively take the limit  $m_H \rightarrow 0$  the string becomes infinitely thick and its tension goes to zero [6]. This means that there are no strings in the limit  $m_H = 0$ . The ANO vortex becomes a vacuum state with "twisted" boundary conditions which ensure the magnetic flux  $2\pi/n_e$ .<sup>9</sup> The absence of ANO strings in theories with flat Higgs potential

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<sup>9</sup>The author is indebted to A. Vainshtein for this interpretation.

was noticed in [33, 34]. ANO strings on the Higgs branch was also discussed in [35] from the brane point of view. However it was not noticed in [35] that the vortex becomes infinitely "thick" and its tension goes to zero.

One might think that the absence of ANO strings means that there is no confinement on Higgs branches. As we will see now this is not the case [6].

So far we have considered infinitely long ANO strings. However the setup for the confinement problem is slightly different [6]. We have to consider monopole–anti-monopole pair at large but finite separation  $L$ . Our aim is to take the limit  $m_H \rightarrow 0$ . To do so let us consider ANO string of the finite length  $L$  within the region

$$\frac{1}{m_\gamma} \ll L \ll \frac{1}{m_H} . \quad (5.20)$$

Then it turns out that  $1/L$  plays the role of the  $IR$ -cutoff in Eqs. (5.16) and (5.19) instead of  $m_H$  [6].

The reason for this is easy to understand. In fact, the reason for the absence of the vortex solution is quite clear from (5.18). If we put  $m_H$  to zero the scalar field  $\varphi$  cannot reach its VEV at infinity because of its logarithmic behavior. This was noticed in [33].

Now consider vortex of finite length  $L$  and look at the behavior of  $\varphi$  at large distances  $r$ ,  $L \ll r \ll 1/m_H$ . In this region the problem becomes three dimensional. The solution of the free equation of motion in three dimensions reads

$$\varphi - v \sim \frac{1}{|x|} , \quad (5.21)$$

where  $|x|$  is three dimensional distance from the vortex. The solution (5.21) perfectly goes to its VEV  $v$  at infinity. To put it in the other way at distances  $|x| \gg L$  from the vortex the two dimensional problem transforms into the three dimensional one. Namely, the scalar field has logarithmic behavior at  $r$  within the bounds  $R_g \ll r \ll L$ , whereas at  $|x| \gg L$  it acquires  $1/|x|$  behavior given by (5.21). We see that  $L$  really plays the role of the  $IR$ -cutoff for the logarithmic behavior of  $\varphi$ . Now we can take the limit  $m_H \rightarrow 0$ .

The result for the electromagnetic core of the vortex becomes [6]

$$R_g^2 \sim \frac{1}{m_\gamma^2} \ln^2 m_\gamma L , \quad (5.22)$$



while its string tension is given by [6]

$$T = \frac{2\pi v^2}{\ln m_\gamma L} . \quad (5.23)$$

We see that the ANO string becomes "thick" but still its transverse size  $R_g$  is much less than its length  $L$ ,  $R_g \ll L$ . As a result the potential between heavy well separated monopole and anti-monopole is still confining but is no longer linear in  $L$ . It behaves as [6]

$$V(L) = 2\pi v^2 \frac{L}{\ln m_\gamma L} . \quad (5.24)$$

As soon as the potential  $V(L)$  is an order parameter which distinguishes different phases of a theory (see, for example, review [36]) we conclude that we have a new confining phase on the Higgs branch of the Seiberg–Witten theory. It is clear that this phase can arise only in supersymmetric theories because we do not have Higgs branches without supersymmetry.

Unfortunately, the confinement on Higgs branches cannot play a role of a model for QCD-like confinement. In particular, the confining potential (5.24) gives rise to the following behavior of Regge trajectories at ultra-large  $j$ ,  $j \gg 1/g^2$  (see (4.18))

$$E^2 \sim v^2 \frac{j}{\ln(g^2 j)} . \quad (5.25)$$

We see that Regge trajectories never becomes linear. At  $j \lesssim 1/g^2$  they are non-linear due to the reason we have discussed in subsection 4.3. At  $j \gg 1/g^2$  they are still non-linear because of the logarithmic factor in (5.25).

It is worth note also that because of type I superconductivity on the Higgs branch we have a tower of "exotic" hadron states corresponding to higher winding numbers of strings.

## 6 Outlook: tensionless strings

We have discussed two confinement scenarios in the Seiberg–Witten theory: the confinement of quarks in the monopole point upon breaking  $\mathcal{N} = 2$  down to  $\mathcal{N} = 1$  and the confinement of monopoles on the Higgs branch in  $\mathcal{N} = 2$  theory. The latter one does not looks like confinement in QCD, while the former one is more promising.

Still as we have discussed in Section 4 it has several unwanted features. Basically these unwanted features fall into two categories. First, is the presence of extra states in the hadron spectrum. It is a reflection of  $U(1)$  nature of the confinement in Seiberg–Witten theory. The second is associated with the large value of the string tension (4.21) as compared with the square of the photon or Higgs mass.

One can believe that the first problem disappears if we increase  $\mu$  and go to the limit of  $\mathcal{N} = 1$  QCD. Now I would like to speculate that the second problem also might disappear in the same limit.

One might think that once we increase  $\mu$  the dual QED coupling  $g^2$  increases and becomes large,  $g^2 \gg 1$ . If this happens the string tension becomes small, see (4.21). As a result Regge trajectories become linear already at  $j \sim 1$ . Moreover, hadrons built of quarks become light and can be visible in the low energy spectrum. Instead monopoles and photon (to be interpreted as glueballs) become relatively heavy. Of course, at strong coupling we lose control over the theory and cannot use eq. (4.21) relating the string tension to the photon mass. The string is not a BPS state and its tension is not protected from corrections. Moreover, we do not have exact formula for the photon mass in the strong coupling as well. Still we can speculate that there is a region in the parameter space where the string tension becomes much less than the square of the photon mass.

In fact, the conjecture that ANO strings might become tensionless in the strong coupling limit of QED was suggested in Ref.[37] in order to find the field theory explanation of tensionless  $M$ -theory strings which arise when 5-branes approach each other.

Note, that we have to find the regime such that  $T$  goes to zero, while  $m_\gamma$  and  $m_H$  (which control the transverse size of ANO string) stay finite.

For example, the string on a Higgs branch discussed in the previous section does not do the job. Its tension goes to zero but its transverse size  $R_g$  becomes infinite (see (5.16)). Thus this string disappears. Another example is the string in the monopole point at the  $AD$ -value of the quark mass (see subsection 4.4). As the monopole condensate vanishes the string tension goes to zero, however the string transverse size might become infinite. It is not clear if this string can serve as a QCD string.

To have a QCD-like confinement we need a regime in which string remains a string (its size does not grow) while its tension becomes small.

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